

### 2.3.2 Models of CAPM

As suggested by portfolio theory, only systematic risk matters to investors but not for nonsystematic risk. The reason is that non-systematic risk is the risk that can be eliminated with no cost through diversification. Thus, the financial market should compensate the systematic risk by a risk premium, but not to nonsystematic risk (Perold, 2004). This implication is summarized in the following CAPM formula:

$$E[r_i] - r_f = \beta_i (E[r_M] - r_f)$$

where

$E[r_i]$  = expected return of individual stock  $i$

$r_f$  = risk free return

$\beta_i$  = beta coefficient of individual stock  $i$

$E[r_M]$  = return of market portfolio, or simply market return

The term “ $E[r_i] - r_f$ ” refers to the risk premium of individual asset  $i$ , which is the amount of expected return of individual asset  $i$  above the risk-free rate. Similarly, the term “ $E[r_M] - r_f$ ” implies risk premium of the market portfolio.

According to the deviation of CAPM, the beta coefficient is the covariance of the individual stock with the market portfolio over the variance of market portfolio. The beta coefficient is a measure on the systematic risk of an individual stock. One can consider two typical cases: the risk-free asset and the market portfolio. The beta coefficient of the risk-free asset is zero, since there is no variation on the price of risk-free asset. The covariance between risk-free asset and market portfolio is zero and so as the beta coefficient of the risk-free asset. For the market portfolio, the covariance between two market portfolio equals to the variance of a market portfolio, and so the beta coefficient of market portfolio is 1 .

If a stock is more correlated with the market, then it would be more volatile and more risky, driven by the market. Therefore, the systematic risk of the stock is larger and the beta coefficient is larger also. Investors would demand for a higher return to compensate for the systematic risk (Sharpe, 1990). Such compensation of return is directly proportional to the level of systematic risk of the stock. The equation can be re-interpreted as:

$$\beta_i = (E[r_i] - r_f) / (E[r_M] - r_f)$$

The formula suggests the expected return, or the required return of an individual stock, is positively related to the beta coefficient as implied by the CAPM (Bodie, Kane and Marcus, 2004). In reality, asset return is not driven by the market factor only, but there are firm-specific factors that affect the asset return also. This issue is summarized in the following CAPM equation:

$$r_{it} - r_{ft} = \beta_i(r_{Mt} - r_{ft}) + \varepsilon_t$$

where

$\varepsilon_t$  = non-systematic risk of the stock, or firm-specific risk of the stock

There are a number of implications that CAPM gives to investment management. The first important implication of CAPM is that all investors select financial assets to optimize portfolios. Ultimately, they arrive the same portfolio as the portfolio is the market portfolio (Sharpe, 1990). CAPM implies that the market portfolio is the most efficient portfolio and all investors should hold it. According to the CAPM assumption, investors are identical and so their final decision on portfolio holding should be the same. An intuitive implication is that identical investors would hold the same portfolio, and each investor should hold an identical share of the market portfolio.

Firstly, from the assumption of CAPM, it is not different to see that investors would hold the same portfolio. As investors are homogenous in their wealth, beliefs and the way they make analysis, one can conclude that investors in the CAPM world would hold homogeneous and identical assets (Ferson, 1995). Since all investors are identical, it is intuitive that they would hold the same portfolio.

The second important implication by CAPM is that the market portfolio is the optimal portfolio. This result confirms the implications of portfolio theory that the

market portfolio is the well-diversified portfolio and be the optimal portfolio to mean-variance optimizer(Gilson & Kraakman, 2002). The market portfolio is the only portfolio that contains on non-systematic risk. Any portfolio that ignores any single asset in the market would lose certain power of diversification, and thus be sub-optimal to the market portfolio.

The third important implication of CAPM is that that passive strategy is efficient. The CAPM world in itself suggests that the financial market is efficient and all information are transparent and available to investors. Passive strategy implies that investors' optimal strategy is to hold the market portfolio. Security analysis or technical analysis or other active strategy would not deliver positive return to investors (Li et al., 2003)..

### **2.3.3 Empirical tests of CAPM**

The tests on CAPM by using real world data is extensive since the emergence of the model in 1960s. Recently, a review on theory and empirical tests of CAPM is summarized by Fama and French (2004), who examine the empirical tests of CAPM since the emergence of the model. The main theme of the empirical tests are to examine how well do CAPM explains the real world financial market, and asks whether CAPM correctly depicts the stock price behavior.

There have been extensive tests on whether CAPM predicts the real world markets correctly in the academic literature. Fama and French (2004) offer a summary for the theory and evidence of CAPM. They examine the empirical

researches of CAPM in the past 30 years. Many empirical tests are carried out to see how well CAPM explains the real financial market. It is noted that the main research method and statistical analysis tools used is regression analysis. In an typical regression analysis, researchers take the approach that use asset return or stock return minus risk-free rate regress on the market portfolio return minus risk-free rate and estimate the beta coefficient.

There are two major results in the empirical tests of CAPM. In the first major result, CAPM predicts that excess returns should be positively related to the systematic risks associated with portfolio. Thus, the intercept of the regression, or in a specific term Jensen's alpha, should be zero statistically (Avramov & Chordia, 2006). However, it is found that a significant alpha is common to stocks in the stock market. In the second major result, researchers tested that the relationship between security return and beta. Researchers found that the beta coefficient is less significant than it should be. Roll's critique is a well-known critique on the empirical research method of CAPM. Roll (1977) suggested that it is impossible to create a market portfolio in the spirit of CAPM. According to Roll, a market portfolio in real world should include all securities, real estates, fixed assets and human capital etc., which is impossible to construct empirically.

An empirical test on Roll's critique is performed by Stambaugh (1982). In the test, the researcher consider two market portfolio. One is a restricted market portfolio that include major stock index only. The other is an expanded market portfolio that include all stocks, bonds, preferred shares, fixed assets and commodities. In the research result of Stambaugh (1982), it is found that the difference in the explanatory power of restricted market portfolio and expanded market portfolio are similar. Further,

Stambaugh (1982) performs an empirical tests on the Roll's critique. The researcher tests the CAPM model with an expanded market portfolio, from major stock index, medium-to-small market index, corporate bonds, government bonds, preferred stocks, real estate and commodities. The result shows that CAPM is not sensitive by expanding the market portfolio proxy. The main reason is that because the major stock market index is too volatile compare with other securities and thus the variation of stock market return in the markets dominates the variation of other market returns, and thus making the contribution of other asset is insignificant. There result shows that Roll's critique is not a big problem empirically and major stock market index is a proxy that is good enough to work in practice.

Another attempt is mad by Chan, Roll and Ross(1986) to identify risk factors of beta in CAPM model empirically. Beta, as the covariance of asset vs. market return over variance of market return and be the source of systematic risk factors to an asset. It remains to be an unexplained object in the framework of CAPM. Their empirical work explored the black box of systematic risk and they find empirical evidence that macroeconomic variables would have impact on stock return in the channel of future cash flows and discount rates. Their work identified four key macroeconomic variables in systematic risk, namely, term structure of interest rate, industrial production, risk premium in the marketor default spread, and unanticipated inflation.

## 2.4 Other Asset Pricing Model

### 2.4.1 Factor Models

The researches in investment management and finance emphasize the theoretical building and empirical testing of asset pricing models. Financial economists research the factors that would have impacts on the expected return a certain asset. The mathematical characterization is made on factor model, including single-factor model or multi-factor model.

The factor model takes an alternative approach to explain the return of assets in investment theory. The factor model suggest that stock return is related to a number of factors. Then, the stock return can be decomposed to these factors correspondingly (Alexander, Sharpe and Bailey, 2001). The generic form of factor model is:

$$\text{Expected return} = b_0 + b_1 \cdot \text{factor 1} + b_2 \cdot \text{factor 2} + \dots + e$$

The factor model is general enough that it can be applied to identify the factors that may have an impact to the return of an asset empirically. For instance, in studying the return of bond, Fama and French (1993) includes the term structure factors of interest rates and default rates to be the factors that can explain bond returns. Sharpe (1982) considers dividend yields and other accounting or financial information on the annual reports of the company to be factors

attributable to return of stock. Based on the rationale of factor model, CAPM can be classified as one form of factor model, which considers the market risk premium as the only factor. CAPM suggests that market risk premium is the only factor that has impacts towards risk premium of individual stocks.

It should be noted that the CAPM can be considered as a particular factor model, namely market models. CAPM predicts that the stock return is only attributed by one factor, which is the market portfolio return (Bodie, Kane and Marcus, 2004, p.303).

Some models consider the alternative of CAPM and open up the model to consider the impacts of more than one factor towards expected return. A number of important models in finance are multi-factor models such as Arbitrage Pricing Model, Fama-French Three Factor Model and Carhart Four Factor Model. The models are explained into details as the following.

#### **2.4.2 Arbitrage Pricing Model**

Ross (1976) acknowledged the importance of various risk factors on asset prices and developed the arbitrage pricing theory (APT). Arbitrage pricing theory is a multi-factor model and the factors in APT are macroeconomic variables that

Ross did understand the importance of a model with multiple risk factors and he developed the arbitrage pricing theory (APT). The APT is a multifactor model with